THEORY
OF
EASUREMENTS

STEVENS

THEORY OF MEASUREMENTS

A MANUAL FOR
PHYSICS STUDENTS

BY

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ILLUSTRATED

SECOND EDITION, REVISED

NEW YORK
D. VAN NOSTRAND COMPANY
25 PARK PLACE
1917

PREFACE

THIS book is designed to be used in either of two ways:

1. *As a Text-book.* The work outlined would require two semester hours for its completion. By extending the discussions and problems, it may be made to cover three semester hours; or by omitting portions of the theory, the student may gain a working knowledge of the subject in a shorter time. A "rule of thumb" knowledge of adjusting observations, however, is not to be recommended.

2. *As a Laboratory Guide.* The work would cover a three years' course in the physical laboratory. During the first year, the student would make use of those portions which are devoted to methods of estimating precision, and the propagation of errors; in the second year the methods of adjustment of observations would be used; and in the third year the student should be prepared to discuss his results by the use of empirical formulae and curves. The work of the second year is well adapted to students in junior courses in engineering, the adjustment of data obtained from surveys being especially appropriate.

The use of the graphic method would be illustrated throughout the entire course.

Considerable space is devoted to the theory of probability. This subject is a fascinating one to students, presumably on account of its human interest.

J. S. S.

University of Maine, Orono, Me.,
January, 1915.

CONTENTS

NOTATION

The following notation will be used in this book:

y = Simple probability;
P = Compound probability;
Q = Negative probability;
m = Single observation;
M = Mean of observations;
z = True value of a component observation;
Z = The resultant true value;
x = Error;
v = Residual;
h = Measure of precision;
k = Probability of error zero;
r = Probable error of a single observation;
r_0 = Probable error of the mean;
δ = Error in a component measurement;
Δ = Error in a result;
a.d. = Average deviation of a single observation;
A.D. = Average deviation of the mean;
p = Weight (Latin *pondus*);
E = Huge error.

THEORY OF MEASUREMENTS

CHAPTER I

INTRODUCTION

LORD KELVIN has told us that one's knowledge of science begins when he can measure what he is speaking about and express it in numbers. Every year a vast number of measurements are made in physical, chemical, and engineering laboratories, as well as in laboratories for advanced research. We are unable, however, to state concerning any one of these measurements that the result is absolutely correct. One of the most precise measurements in physical science is that of the wave-length of light. The wave-length of cadmium light, measured by a Michelson interferometer and a Rowland grating, was found to be

$$\lambda_c = 0.000064384722 \text{ cm. (Michelson)}$$
$$\lambda_c = 0.00006438680 \text{ cm. (Rowland).}$$

Or we may put it in another way,

1 meter contains 1553163.6 wave-lengths (Michelson),
1 meter contains 1553164.1 wave-lengths (Fabry and Perot).

These measurements were made by different observers using different methods. They are remarkable for their agreement and they give us the wave-length of light with sufficient accuracy for all purposes. But they are not correct, and it is not at all likely that we shall ever know the true length of a wave of cadmium light.

It is of great importance, however, that we should be able to pass judgment upon the accuracy of measurements like these. To borrow another illustration from the domain of optics, we may assume the value for the velocity of light to be 299860 ± 30 km. per second. Here the number 30 is a measure of precision, which, if omitted, would make the statement ridiculous, since the velocity of light has not been measured with sufficient accuracy to be regarded as correct to a single kilometer.

A student who brings in a value for g, resulting from measurements with a simple pendulum, of 982.436 centimeters per second per second, is likely to look upon the result with complacency until he is made to see that all figures after the 8 are useless because erroneous.

MEASUREMENTS

Measurements are usually classified as follows:

1. Direct—when, for example, a distance is measured with a tape line.

2. Indirect—when, for example, the density of a cylinder is determined by measurements of its length, diameter, and mass.

3. Conditioned—when, for example, the third angle of a triangle is restricted by the values of the other two angles.

Measurements not so conditioned are called *independent*.

ERRORS

Errors in measurement may be divided into two general classes:

1. Those which may be eliminated, in part at least, by improving adjustments and taking greater care in the method employed.

These include **errors in instruments,** such, for example, as are caused by faulty measuring sticks, imperfectly graduated circles, and poorly adjusted balance beams. The obvious method of correcting these errors is to substitute good instruments for poor ones; or, when possible, to eliminate the errors by compensation.

Personal Errors. These are errors characteristic of the individual observer. If we swing a pendulum before a class of students and ask them to indicate the times of greatest displacement by tapping with their pencils, the result will well illustrate the personal equation. In the experiment in wireless telegraphy by which time messages were sent from Paris to Arlington, each observer was carefully rated in order that this error should be guarded against.

Mistakes are, unfortunately, too common in students' laboratory work to call for an extended explanation. Errors in reading scales, in computation, and in tabulation may be classed as mistakes, and these should become less and less frequent as the student gains experience.

2. The second class consists of errors which are indeterminate in their nature, and may not be entirely eliminated, however much care we may take in our measurements, or by the use of the highest grade of apparatus. It is these errors with which we are concerned in the discussions in this book.

CHAPTER II

PROBABILITY

PROFESSOR JEVONS in his *Principles of Science* states a truism which has an important bearing upon the theory of probability. "Perfect knowledge alone can give certainty, and in Nature perfect knowledge would be infinite knowledge, which is clearly beyond our capacities. We have, therefore, to content ourselves with partial knowledge—knowledge mingled with ignorance producing doubt."

We may interpret this to mean that from the point of view of Omniscience everything exists as certainty. The path of a leaf falling from a tree, as well as that of a mote dancing in a sunbeam is known with as great a certainty as that of a heavy body dropped to the earth. For finite minds, however, where certainty is impossible, the ability to pass upon the probability that an event will happen in a certain way is the best substitute attainable.

What is highly probable in minds of a certain order of intelligence, may be improbable to others. To a trained meteorological observer, it may appear extremely probable that it will rain to-morrow; to one who bases his weather predictions upon popular superstition, it may be improbable. Again it may seem probable to certain people that spirit hands produced music at a séance, while to others the probability becomes negative.

We may look at the subject in another way. When a championship game of baseball has been finished, the result becomes a certainty in the minds of the spectators; while in various parts of the country, where the result has not as yet been reported, bets continue to be made expressing the probability of what to other minds is a certainty.

If a coin is tossed up under normal conditions, the probability that it will fall "heads" is one out of two, or one-half. This is also the probability that it will fall "tails." We have, then, from our notation, $y=\frac{1}{2}$, or, since we shall deal with more than one event, $P=\frac{1}{2}$ and $Q=\frac{1}{2}$. Since the coin must fall in one way or the other, we have

$$P+Q=1 \text{ (the symbol for certainty).}$$

If n coins are thrown, we have, by the binomial theorem

$$(P+Q)^n=P^n+nP^{n-1}Q+\frac{n(n-1)}{\underline{|2}}P^{n-2}Q^2+\ldots$$

The first term expresses the probability that all will come down heads, the second that all but one will come down heads, etc.

If we take $n=6$, the chances that all will be heads may be expressed by the fraction $\frac{1}{64}$, and that five will be heads by $\frac{6}{64}$.

PROBLEMS

1. By expanding the binomial to the proper number of terms, show that the chances for four, three, two, one, and no heads may be expressed by $\frac{15}{64}$, $\frac{20}{64}$, $\frac{15}{64}$, $\frac{6}{64}$, and $\frac{1}{64}$.

2. By using an additional term prove that it is impossible to throw seven heads with six coins.

3. Plot a curve with the data obtained, and preserve it for future use.

4. If the class is sufficiently large (say 20) it will prove an interesting exercise to take the mean results of 64 trials by each student and compare with the results obtained by theory.

A better acquaintance with the laws of probability may be obtained by putting aside the formula and solving the following problems by an appeal to reason.

PROBLEMS

1. If two dice are thrown what is the probability that the sum of the numbers will be five? We first determine the possible results with two dice, which may be obtained by considering that each number on one die may appear with each number on the other. This gives us $6\times 6=36$. Five may be obtained either by a four and a one, or a two and a three. Now the four may be on the first die and the one on the second, or the one may be on the first and the four on the second. The same is true for the two and three. Thus we have four possibilities, giving a probability of $\frac{4}{36}$.

2. What is the probability of throwing one ace with a single die in one throw?

3. What is the probability of throwing no ace with a single die in one throw?

4. What is the probability of throwing one ace in two trials?

5. What is the probability of throwing two aces in two trials?

6. What is the probability of throwing no ace in two trials?

7. What is the probability of throwing only one ace in two trials?

8. A bag contains eight red, six black, and five green balls. What is the probability of drawing first a red and then a black ball in two trials?

9. The class record shows that each student in a class of twenty-five usually solves one problem out of three assigned. What is the probability that an assigned problem will be solved?

10. In Merriman's *Least Squares* we have this problem: Let a hundred coins be thrown up each second by each of the inhabitants of the earth. How often will a hundred heads be thrown in a million years? (It will prove interesting for the student to guess the answer to this problem before solving it. The number of inhabitants may be taken as one and one-half billion.)

Before leaving the subject of probability a few general considerations may be suggested. It is a rather prevalent notion that antecedent happenings have an effect upon present probability. If a coin has come down heads five times in succession, it is pretty difficult to convince the average man that the chances for tails on the sixth throw are not greater than the chances for heads. Of course, if the conditions are normal, the probability of throwing heads is just one-half. (The derivation of the word *chance* is an interesting one.) Such expressions as "the turning

of luck" indicate the strong hold this feeling has upon the majority of people.

Again, we should not become over-confident from the results of the laws of probability. Interesting applications may be found in connection with the periodic law of Mendeleéff; the law of Prout, which states that the atomic weights of the other elements are exact multiples of that of hydrogen; and the kinetic theory of gases as developed by Clausius and Mayer. A case in which the probability almost becomes a certainty is illustrated by the coincidence of the seventy spectral lines in iron vapor with those in solar light. The possible arrangement of the seventy lines would be

$$70 \times 69 \times 68 \times \ldots 4 \times 3 \times 2 \times 1.$$

But as all possible arrangements would not apply, the probability of a chance coincidence has been estimated by Kirchhoff as one in one trillion. Do we know that iron exists in the sun?

The following illustration may serve to weaken our feeling of certainty: Imagine a fly watching two coincidence pendulums which come together on the eighty-first swing. While we are about it, let us imagine that in the mind of a fly, one second represents a year. The fly will watch the vibrations for ten seconds (years) and report to another fly which has just come up, that quite an extended series of observations fails to discover the two pendulums in coincidence. The pendulums are watched through fifty, sixty, and seventy swings and it becomes a

law in flyland that pendulums do not get together. When the law has become established by observations extending over eighty fly-years, the bell rings and the assumption based upon a very reasonable law of probability breaks down.

The Probability Curve. Referring to problem 3, page 6, it will be seen that the curve takes the form indicated in Fig. 1.

Any similar data from experiments which follow the laws of probability would yield such a curve as this.

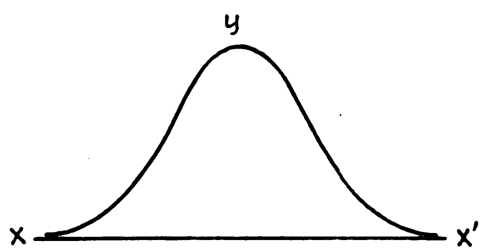

FIG. 1.

A familiar illustration is afforded by target practice. If the target is marked off into divisions, the distances of these divisions from the center may be taken to represent the magnitude of the errors. If a series of shots be fired by expert marksmen under normal conditions, they will form the basis for a curve which will resemble Fig. 1.

Since the errors are plotted along the x-axis and their corresponding probabilities along the y-axis; and since the curve is seen to be symmetrical with respect to the

y-axis, and the x-axis is an asymtote, it will be seen that its equation must take some such form as

$$y=e^{-x^2}.$$

A rigid deduction of this probability equation has been given by Gauss.

Let x represent an error and y its probability. Then

$$y_1=f(x_1), \quad y_2=f(x_2), \ \ldots \ y_n=f(x_n). \quad . \quad . \quad . \quad . \quad (1)$$

For compound probability

$$P=y_1y_2 \ \ldots \ y_n=f(x_1)f(x_2) \ \ldots \ f(x_n); \quad . \quad . \quad (2)$$

$$\log P=\log f(x_1)+\log f(x_2)+ \ \ldots \ \log f(x_n).$$

Let us suppose that n observations are taken on the quantities z_1, and z_2. The most probable values of these quantities will make P maximum,

$$\frac{dP}{Pdz_1}=\frac{df(x_1)}{f(x_1)dz_1}+\frac{df(x_2)}{f(x_2)dz_1}+ \ \ldots \ \frac{df(x_n)}{f(x_n)dz_1}=0 \quad . \quad . \quad (3)$$

$$\frac{dP}{Pdz_2}=\frac{df(x_1)}{f(x_1)dz_2}+\frac{df(x_2)}{f(x_2)dz_2}+ \ \ldots \ \frac{df(x_n)}{f(x_n)dz_2}=0 \quad . \quad . \quad (4)$$

We have

$$\left.\begin{aligned} df(x_1)&=\phi(x_1)f(x_1)dx_1 \\ df(x_2)&=\phi(x_2)f(x_2)dx_2 \\ df(x_n)&=\phi(x_n)f(x_n)dx_n \end{aligned}\right\} . \quad . \quad . \quad . \quad . \quad (5)$$

By differentiating $\log f(x_1)$ with respect to x_1, we have

$$\frac{df(x_1)}{f(x_1)dx_1}=\phi(x_1). \quad \text{Or} \quad df(x_1)=\phi(x_1)\, f(x_1)dx_1.$$

Substituting (5) in (3) and (4), we have

$$\phi(x_1)\frac{dx_1}{dz_1}+\phi(x_2)\frac{dx_2}{dz_1}+\ldots\phi(x_n)\frac{dx_n}{dz_1}=0 \quad . \quad . \quad (6)$$

$$\phi(x_1)\frac{dx_1}{dz_2}+\phi(x_2)\frac{dx_2}{dz_2}+\ldots\phi(x_n)\frac{dx_n}{dz_2}=0 \quad . \quad . \quad (7)$$

The equations may be simplified as follows: Consider that z has been measured n times, giving $m_1, m_2, \ldots m_n$ as results. Then

$$\begin{array}{c} z_1-m_1=x_1 \\ z_1-m_2=x_2 \\ \hline z_1-m_n=x_n \end{array}$$

Since m_1, m_2, and m_n are constants, we have

$$\frac{dx_1}{dz_1}=1; \quad \frac{dx_2}{dz_1}=1; \quad \frac{dx_n}{dz_1}=1; \quad . \quad . \quad . \quad . \quad (8)$$

and similar results follow for dz_2.

This greatly simplifies Eqs. (6) and (7) and produces

$$\phi(x_1)+\phi(x_2)+\ \ldots\ \phi(x_n)=0. \quad . \quad . \quad . \quad (9)$$

Errors and Residuals

We must now make a brief digression in order to illustrate the difference between errors and residuals. Take the following measurements, of equal weight:

$$
\begin{aligned}
m_1 &= 430.6 \\
m_2 &= 429.9 \\
m_3 &= 431.1 \\
m_4 &= 430.8
\end{aligned}
$$

By universal custom the mean of these results, 430.6, is taken as the best attainable value. (Later a mathematical proof of this will be given.)

The differences between the measurements and the mean are called residuals, and are as follows:

$$
\begin{aligned}
v_1 &= 0.0 \\
v_2 &= -0.7 \\
v_3 &= +0.5 \\
v_4 &= +0.2
\end{aligned}
$$

The algebraic sum of the residuals always equals zero, and this may serve as a check upon the work. Now if we had some way of knowing that 430.6 was the correct result, we would transform those residuals into errors and designate them by x_1, x_2, x_3, and x_4.

A residual, then, is the difference between a measurement and the best attainable result; an error is the difference between a measurement and the true result.

It is obvious, first, that we cannot determine the values of the errors; and secondly, that the sum of the errors will approach the sum of the residuals as we increase the number of measurements.

Going back to Eq. (9), we may assume that n is sufficiently large so that we can apply the above law. Then, since the sum of the residuals is always zero, one may take the sum of the errors to be zero, and write

$$x_1+x_2+\ \ldots\ x_n=0.$$

From this it follows that ϕ is a constant, which we may call c.

If we substitute the values of $\phi(x_1)$, $\phi(x_2)$, and $\phi(x_n)$ for (5) in (9), we obtain

$$\frac{df(x_1)}{f(x_1)dx_1}+\frac{df(x_2)}{f(x_2)dx_1}+\ \ldots\ \frac{df(x_n)}{f(x_n)dx_1}=cx_1+cx_2+\ \ldots\ cx_n.$$

Since this holds for any number of observations, the corresponding terms are equal. Omitting subscripts we have in general

$$\frac{df(x)}{f(x)dx}=cx.$$

Substitute values from (1) and integrate. We have

$$\log y=\frac{cx^2}{2}+k',$$

$$y=e^{\frac{1}{2}cx^2}e^{k'}.$$

c is negative (why?) and may be replaced for the sake of convenience by $-2h^2$. We may also replace $e^{k'}$ by k. Then $y = ke^{-h^2x^2}$, which is the equation of the probability curve. It is similar in form to the equation suggested to express the curve in Fig. 1.

This is the most important equation in the theory of precision of measurements and its meaning should be clearly understood.

PROBLEMS

1. Show from the curve that positive and negative errors are equally likely to occur.
2. Show that k represents the probability of the error zero.
3. Explain why h is called the measure of precision.
4. Show that the curve is horizontal over the origin.
5. Show that a point of inflection occurs when

$$x = \pm\frac{1}{h\sqrt{2}}.$$

THE PROBABILITY INTEGRAL

In order to express the probability that a certain group of errors will be made we integrate between the limits concerned. As this involves compound probability, we have

$$P = \frac{k}{dx}\int_{x_2}^{x_1} e^{-h^2x^2}dx.$$

For certainty $P=1$ and the limits become $+\infty$ and $-\infty$.

$$1 = \frac{k}{dx}\int_{-\infty}^{+\infty} e^{-h^2x^2}dx.$$

This is a well-known integral and is discussed in various treatises on the calculus. It equals $\frac{\sqrt{\pi}}{h}$, and therefore

$$1=\frac{k\sqrt{\pi}}{dxh}, \qquad k=\frac{hdx}{\sqrt{\pi}},$$

and the probability equation becomes

$$y=hdx\pi^{-\frac{1}{2}}e^{-h^2x^2}.$$

For compound probability

$$P=\frac{h}{\sqrt{\pi}}\int_{x_2}^{x_1}e^{-h^2x^2}dx.$$

The probability that an error lies between $+x$ and $-x$ is double the probability that it lies between x and 0. We have

$$P=\frac{2h}{\sqrt{\pi}}\int_0^x e^{-h^2x^2}dx.$$

This may take the form

$$P=\frac{2}{\sqrt{\pi}}\int_0^{hx} e^{-h^2x^2}d(hx).$$

This is the usual form of the probability integral. h may be determined in a manner to be explained later; then by use of a table the probability of any desired magnitude may be obtained. The student is referred to any of the larger text-books on *Least Squares* for illustrations.

EXERCISES

In order to become familiar with the terms h and k, the following curves should be plotted:

1. Consider h constant. Give k the values 1, 2, 3, 4. Plot a curve for each value.

2. Consider k constant. Give h the values $\frac{1}{4}$, $\frac{1}{2}$, 1, 2.

In each case select a series of rather small numbers for x and use the formula

$$y=ke^{-h^2x^2}.$$

3. If time permits, a set of curves should be constructed in which both h and k are variables, x and y having constant values.

The Term "Least Squares"

If we take the product of a number of single probabilities, we obtain

$$P=y_1y_2\ .\ .\ .\ y_n=k^ne^{-h^2(x_1^2+x_2^2+\ .\ .\ .\ x_n^2)}.$$

An inspection of this equation shows us that the probability is greatest when the expression in the parenthesis is least. But these numbers are the squares of the errors (residuals, see page 12). It follows that the most probable values of observed quantities are those which make the sum of the squares of the residuals the least. From this law is derived the term Least Squares.

THE ARITHMETICAL MEAN

We are now prepared to prove that in a set of observations, the arithmetical mean has the greatest probability.

Let M be the mean of observations $m_1, m_2 \ldots m_n$. Then

$$M-m_1, \quad M-m_2 \ldots M-m_n \text{ are residuals.}$$

We may make the sum of their squares a minimum.

$$\frac{d}{dM}[(M-m_1)^2+(M-m_2)^2+\ldots (M-m_n)^2]=0.$$

$$2(M-m_1)+2(M-m_2)+\ldots 2(M-m_n)=0.$$

$$M=\frac{m_1+m_2+\ldots m_n}{n}, \text{ the arithmetical mean.}$$

A CONSTANT INTERVAL. FORMULA

$$x=6\frac{(n-1)(m_n-m_1)+(n-3)(m_{n-1}-m_2)+\ldots}{n(n^2-1)}$$

For a constant interval the arithmetical mean should not be employed. This formula comes from formulae used in developing normal equations. See Mellor's *Higher Mathematics*, page 327.

If n is odd, the middle term does not appear. Use an even number of measurements.

ILLUSTRATIVE PROBLEMS

1. Kundt's experiment: 30.7, 43.1, 55.6, 67.9, 80.1, 92.3, 104.6, 116.9, 129.2, 141.7, 154.0, 166.1 cm. Ans. 12.3 cm.

2. Time of vibration of magnet bar: 3.25, 9.90, 16.65, 23.35, 30.00, 36.65, 43.30, 50.00, 56.70, 63.30, 69.80, 76.55, 83.30, 89.90, 96.65, 103.15, 109.80, 116.65, 123.25, 129.95, 136.70, 143.35. Ans. 6.67.

3. A problem illustrating this method may easily be suggested from measurements with a planimeter; also from the acceleration curve with a dropped fork.

WEIGHTS

Heretofore we have considered all our measurements to be of equal value. Suppose the length of a small object was measured with a micrometer gauge, a vernier reading to hundredths of a millimeter, a vernier reading to tenths of a millimeter, and a meter stick.

The following results might be recorded:

1. Micrometer..................	3.542 cm.
2. Vernier *A*....................	3.544 "
3. Vernier *B*....................	3.54 "
4. Meter Stick..................	3.55 "

The mean of these measurements, 3.544, is obviously not the best value, since some represent greater precision than others. If one attaches to these results a number indicating their relative values, such a number is called the

weight and is usually designated by p. (Latin, *pondus*, weight. Compare pound.) Further on, rules will be given for finding the weight of observations, but for the present we may use our judgment. No. 4 is evidently the least accurate and we may give it the weight 1. No. 3 comes next and we may call it 3. No. 1 and No. 2 are about alike in weight, with perhaps a little advantage in favor of No. 1. We may, therefore, assign 8 to No. 2 and 10 to No. 1. Putting our results in tabular form, we have:

No.	Obs. (m)	Weights (p)	Weighted Obs. (pm)
1	3.542	10	35.420
2	3.544	8	28.352
3	3.54	3	10.62
4	3.55	1	3.55

$$\Sigma p = 22 \qquad \Sigma pm = 77.942$$
$$\Sigma pm \div \Sigma p = \text{weighted mean} = 3.543$$

The weighted mean may be proved to have the greatest probability in a manner similar to that employed for the unweighted mean.

PROBLEMS

1. In establishing a north and south line, the following readings were taken:

N. 6′ E., N. 4′ W., N. 50″ W., N. 00′.

All readings were taken with equal care, but the first was taken twice and the third three times. Find the best value.

2. Joule's values of the mechanical equivalent of heat have been weighted by Rowland as follows:

442.8 (0);	427.5 (2);	426.8 (10);
428.7 (2);	429.1 (1);	428.0 (1);
425.8 (2);	428.0 (3);	427.1 (3);
426.0 (5);	422.7 (1);	426.3 (1).

He concludes that 426.9 best represents the result of Joule's work.

3. The sum of the angles of an equilateral triangle is found to measure 180° 9′. If the first angle has been measured six times, the second three times, and the third once, how should the error be distributed among the angles?

Additional problems in weighting will be given after the subject of precision of measurement has been discussed.

Subjects for Discussion

The following topics are suggested for those who may wish to pursue the subject further:

1. The part played by the theory of probability in the work of the U. S. Weather Bureau.

2. The bearing of the laws of probability upon popular superstitions. Read the Vice-Presidential address of Professor A. G. Webster, at Atlanta, Ga., Dec. 30, 1913. (*Science*, Jan. 9, 1914.)

3. Exceptional phenomena. Read Chapter 29 in Jevons's *Principles of Science*.

4. Relation to gambling.

Dr. H. G. Burnham, of Chicago, thinks that the best way to wipe out gambling in America is to teach the children in the schools the laws of chance. He feels sure that the result of this would be that in childhood they would steer clear of the slot machine, and that when they grow up, they would shun the book maker and every other gambling magnate.

To quote further: "The ordinary gambler does not stop to count his chances when he sees that by betting a dollar he may win one hundred dollars. If he had been taught in school to see that in reality the chances were 200 to one against him, and that he was betting a dollar against fifty cents, he would keep his money in his pocket."

5. The use of the theory of probability in mortality tables.

CHAPTER III

THE ADJUSTMENT OF OBSERVATIONS

IT IS a frequent experience in making measurements that our results do not check. The measurements may be conditioned (see page 2). The sum of the angles of a triangle, for example, may not come out just 180°. Or they may be measurements which have a less degree of interdependence.

If we take O as the starting-point and measure a distance, $m_1=6.2$ feet above O; then $m_2=4.3$ above m_1; then $m_2=10.6$ above O; we observe that there is a discrepancy somewhere. In this case, an obvious method of adjustment would be to add 0.03 to the first and second observations and subtract it from the third. Since the discrepancy is 0.1, this adjustment would correct it to 0.01.

Theory. While this method would answer very well for the simple case in question, it would break down when applied to more extended measurements. The theory of the method of adjustment is as follows:

Suppose we have unknown quantities $m_1, m_2 \ldots m_n$, and suppose n measurements are made upon these quan-

tities. Then if $a, b, c, \ldots l$ are known constants and M the resulting measured quantity, we have

$$\begin{array}{l} a_1m_1+b_1m_2+\ldots l_1m_k=M_1 \\ a_2m_1+b_2m_2+\ldots l_2m_k=M_2 \\ \hline a_nm_1+b_nm_2+\ldots l_nm_k=M_n \end{array}$$

Since there are more equations than there are unknowns it follows that in general no system of values will exactly satisfy them. Each equation has a most probable value for its unknown terms, but in each case there will be left a small residual v. We may write the equation:

$$\begin{array}{l} a_1m_1+b_1m_2+\ldots l_1m_k-M_1=v_1 \\ a_2m_1+b_2m_2+\ldots l_2m_k-M_2=v_2 \\ \hline a_nm_1+b_nm_2+\ldots l_nm_k-M_n=v_n \end{array}$$

For simplicity we may designate all the terms independent of m_1 by K.

$$\begin{array}{l} a_1m_1+K_1=v_1, \text{ and by symmetry} \\ a_2m_1+K_2=v_2 \\ \hline a_nm_1+K_n=v_n \end{array}$$

Square both sides of these equations and add:

$$(a_1m_1+K_1)^2+(a_2m_1+K_2)^2+\ldots (a_nm_1+K_n)^2 \\ =v_1^2+v_2^2+\ldots v_n^2.$$

The greatest probability occurs when the right-hand member is a minimum. Placing the first derivative equal to zero, we have

$$a_1(a_1m_1+K_1)+a_2(a_2m_1+K_2)+\ .\ .\ .\ a_n(a_nm_1+K_n)=0.$$

Similar equations may be written for $m_2\ .\ .\ .\ m_n$. These are called " normal equations " and their solution gives the most probable values of the observations under consideration.

Rules. We may now summarize the methods of adjusting observations as follows:

1. Write an observation equation for each observation.
2. Form a normal equation for each unknown by multiplying each observation equation by the coefficient of the unknown in that equation and adding the results.
3. Solve the normal equations by any method. These results are the most probable values.

After these " best values " have been found, the residuals should be computed, and the sum of their squares (Σv^2) found. This value is less than that for any other possible values of the residuals.

ILLUSTRATIVE PROBLEM

1. Given the following observation equations,

$$\begin{aligned}3m_1-4m_2+m_3&=-3\\ m_1+2m_2+2m_3&=25\\ m_2+m_3&=10\\ m_1+m_2+m_3&=16\end{aligned}$$

Find the best values and the sum of the squares of the residuals.

We first find the normal equations by the rule. For m_1,

$$\begin{aligned}
9m_1-12m_2+3m_3&=-\ 9\\
m_1+\ 2m_2+2m_3&=\ 25\\
m_1+\ \ m_2+\ m_3&=\ 16\\
\hline
11m_1-\ 9m_2+6m_3&=\ 32
\end{aligned}$$

For m_2,

$$\begin{aligned}
-12m_1+16m_2-4m_3&=12\\
2m_1+\ 4m_2+4m_3&=50\\
m_2+\ m_3&=10\\
m_1+\ \ m_2+\ m_3&=16\\
\hline
-\ 9m_1+22m_2+2m_3&=88
\end{aligned}$$

For m_3,

$$\begin{aligned}
3m_1-4m_2+\ m_3&=-\ 3\\
2m_1+4m_2+4m_3&=\ 50\\
m_2+\ m_3&=\ 10\\
m_1+\ m_2+\ m_3&=\ 16\\
\hline
6m_1+2m_2+7m_3&=\ 73
\end{aligned}$$

Grouping the normal equations,

$$\begin{aligned}
11m_1-\ 9m_2+6m_3&=32\\
-\ 9m_1+22m_2+2m_3&=88\\
6m_1+\ 2m_2+7m_3&=73
\end{aligned}$$

Solving,

$$m_1 = 6.00$$
$$m_2 = 6.13$$
$$m_3 = 3.26$$

To obtain the residuals, we substitute these values in the observation equations,

$$\begin{array}{rrr}
18-24.52+3.26= & -\ 3 & v_1=\ \ 0.26 \\
6+12.26+6.52= & 25 & v_2=-0.22 \\
6.13+\ 3.26\qquad = & 10 & v_3=\ \ 0.61 \\
6+\ 6.13+3.26= & 16 & v_4=\ \ 0.61 \\
\Sigma v^2=0.860 & &
\end{array}$$

It would be an interesting exercise to try other values for m_1, m_2, and m_3 and compare the value of Σv^2 resulting, with the above values. If, for example, we choose $m_1=6$, $m_2=6$, $m_3=3$, it is obvious that the second residual is unity, which alone is greater than the value of Σv^2.

The following observations were made on a triangle:

Angle $A=45°$; angle $B=80°$; angle $C=54°$. $A+B=126°$; $A+C=100°$; $A+B+C=180°$. Find best results.

The observation equations are

$$m_1=45$$
$$m_2=80$$
$$m_3=54$$
$$m_1+m_2=126$$
$$m_1+m_3=100$$
$$m_1+m_2+m_3=180$$

The student should construct the normal equations and find the best values for m_1, m_2, and m_3.

MEASUREMENTS OF A LINE

$A \quad B \quad C \quad D \quad E \quad F \quad G$

FIG. 2.

$AB=4.0$ units	$FG=7.8$ units
$BC=5.1$	$AC=9.0$
$CD=6.9$	$BD=11.9$
$DE=2.0$	$DF=8.2$
$EF=6.1$	$AG=32.0$

Find the best values of the above measurements made along a line.

Here we have 10 observation equations which will reduce to six normal equations containing six unknown quantities, which may be solved in the usual way. The problem is inserted for the purpose of illustration; its solution is hardly worth while.

FORMULAE

When the number of equations is large and the constants are not whole numbers, it saves time and affords a check upon the work to use the following formulae:

$$aa=a_1^2+a_2^2+\dots a_n^2$$
$$ab=a_1b_1+a_2b_2+\dots a_nb_n$$
$$al=a_1l_1+a_2l_2+\dots a_nl_n$$
$$bb=b_1^2+b_2^2+\dots b_n^2$$
$$aM=a_1M_1+a_2M_2+\dots a_nM_n$$

The normal equations become

$$\begin{array}{l} aam_1+abm_2+\ .\ .\ .\ alm_k=aM \\ bam_1+bbm_2+\ .\ .\ .\ blm_k=bM \\ \hline alm_1+blm_2+\ .\ .\ .\ llm_k\ =lM \end{array}$$

The following example will illustrate the use of these formulae:

Observation Equations

$$6m_1-3m_2=15.1$$
$$2m_1+2m_2=13.9$$
$$m_1-2m_2=2.1$$

$$aa=+36+4+1=+41$$
$$ab=-18+4-2=-16$$
$$bb=+9+4+4=+17$$
$$aM=+90.6+27.8+2.1=+120.5$$
$$bM=-45.3+27.8-4.2=-21.7$$

$$41m_1+16m_2=120.5$$
$$16m_1+17m_2=-21.7$$

Let the student compare these results with those found without the use of the formula.

PROBLEMS

1. Adjust the following values and find Σv^2:

$$
\begin{aligned}
m_1 &= +3.06 \\
m_2 &= -1.30 \\
m_1+m_2 &= +1.75 \\
2m_1+m_2 &= +4.81
\end{aligned}
$$

2. Heretofore it has been assumed that all observations have equal weights. If the weights are unequal each observation equation should be multiplied by the square root of its weight and the normal equation then formed. In the preceding example let the first and the second measurements have weights of one each, the third a weight of four, and the fourth a weight of nine, and find the adjusted values.

3. The following observations are taken from a student's field book:

$$
\begin{aligned}
M &= 147^\circ\ 14'\ 04.5'' \\
N &= \ \ 88\ \ \ 58\ \ 06.1 \\
O &= \ \ 93\ \ \ 51\ \ 26.7 \\
P &= \ \ 29\ \ \ 56\ \ 16.6 \\
M+N+O+P &= 359\ \ \ 59\ \ 55.0 \\
M+N+O &= 330\ \ \ \ \ 3\ \ 39.2 \\
N+O+P &= 212\ \ \ 45\ \ 53.0 \\
O+P+M &= 271\ \ \ \ \ 1\ \ 57.8 \\
P+M+N &= 266\ \ \ \ \ 8\ \ 30.0
\end{aligned}
$$

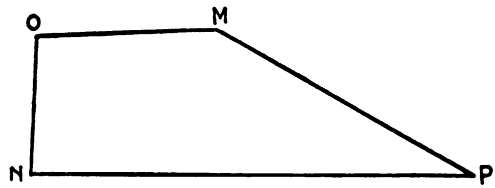

FIG. 3.

Find the best values of the angles.

4.

$$
\begin{aligned}
A &= 22^\circ\ 13'\ 59''\\
B &= 100\ \ 18\ \ 40\\
C &= 57\ \ 27\ \ 33\\
A+B &= 122\ \ 32\ \ 41\\
B+C &= 157\ \ 46\ \ 10\\
C+A &= 79\ \ 41\ \ 29\\
A+B+C &= 180\ \ 00\ \ 3
\end{aligned}
$$

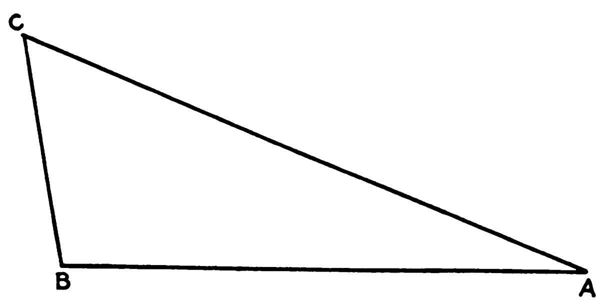

FIG. 4.

Find the best values of the angles.

5. Adjust the following angles:

$BOA = 1$
$BOC = 2$
$COD = 3$
$DOE = 4$
$EOA = 5$
$BOE = 6$

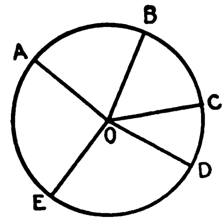

Fig. 5.

No.	Observed Angles.
1.	70° 20′ 51″
2.	52 35 10
3.	50 41 25
4.	95 10 41
5.	91 11 57
6.	198 27 14

Observe that angle 6 = angle 2 + angle 3 + angle 4.

6. Clairaut's empirical formula for the relation between the length of a seconds pendulum and the latitude is

$$l = L_0 + A \sin^2 L.$$

Suppose the following observations to be made:

L	l
0° 0′	0.990564
18 27	0.991150
48 24	0.993867
58 15	0.994589
67 4	0.995325

Substitute the values in the given equation for observation equations; then form normal equations for L_0 and A. Mellor gives these normal equations:

$$0.993099 = L_0 + 0.44765A,$$
$$0.994548 = L_0 + 0.70306A.$$

Systematic Errors

The above method of treatment applies only to accidental errors. If there should happen to be a systematic error running through the problem it is difficult or impossible to detect it from the observations themselves. There may be a mistake on the part of the observer or the error may be due to inexact graduation of the instruments, or it may be due to personal peculiarities. (See page 3.) It is beyond the scope of this book to present the theory of systematic errors, but the following problem may be read over with profit:

$$\begin{aligned}
B &= 86^\circ\ 45'\ 25.2'' \\
A &= 85\ \ 48\ \ 36.4 \\
C &= 81\ \ 16\ \ 40.9 \\
D &= 106\ \ 11\ \ 17.5 \\
A+B+C+D &= 360\ \ \ 2\ \ 00 \\
A+B &= 172\ \ 34\ \ 10.6 \\
B+C &= 168\ \ \ 2\ \ 15.1 \\
C+D &= 187\ \ 28\ \ 13.8 \\
D+A &= 191\ \ 59\ \ 11.3
\end{aligned}$$

The solution according to the method above outlined gives

$$
\begin{aligned}
m_1 &= 85^\circ\ 48'\ 26.2''\\
m_2 &= 86\ \ 45\ \ 32.2\\
m_3 &= 81\ \ 16\ \ 50.1\\
m_4 &= 106\ \ 11\ \ \ 9.5
\end{aligned}
$$

These are the best values, but it will be noted that the sum of these values is $1'\ 58''$ greater than 360°. Presumably, a systematic error of about $30'$ runs through the observations. These values, then, may be further adjusted by finding the weight of each observation and distributing the discrepancy inversely as the weights. The probable error of each observation is given by the formula

$$R = 0.6745\sqrt{\frac{\Sigma v^2}{n-q}},$$

Where n is the number of observations and q the number of unknown quantities. In this case $n-q$ would be 5.

Σv^2 is obtained in the usual manner and found to be 1303. Next we may proceed to find the weights of the various measurements. If we wish to find the weight of m_3, for example, we should represent the absolute term in the normal equation of m_3 by some constant and place the other absolute terms equal to zero. We solve for m_3 and the weight is found to be the reciprocal of the coefficient of the constant. The probable error of m_3 is found by dividing the probable error of each observation by the square root of the weight just found. The discrepancy in the final result may now be adjusted according to the weights found, and should give

$$
\begin{aligned}
m_1 &= 85^\circ\ 47'\ 56.7'' \\
m_2 &= 86\ \ 45\ \ 2.7 \\
m_3 &= 81\ \ 16\ \ 20.6 \\
m_4 &= 106\ \ 10\ \ 40.0
\end{aligned}
$$

The sum of the angles is now found to be exactly 360°. By the principle of least squares the sum of the squares of the residuals is greater than for the "best values" previously obtained. This seems to confirm the suggestion that a constant error ran through the observations. A full discussion of this subject is given in Palmer's "Theory of Measurements," page 117.

SHORTER METHODS

The following abbreviated methods are sometimes employed:

Mayer's Method

Make all the coefficients of m_1 positive and add the results to form a normal equation for m_1. Similarly for $m_2 \ldots m_n$. Solve the normal equations as before.

The results are not so accurate as those obtained by the longer method, but are satisfactory for most purposes.

PROBLEMS

Observation equations.

$$
\begin{aligned}
x-y+2z &= 3 \\
3x+2y-5z &= 5 \\
4x+y+4z &= 21 \\
-x+3y+3z &= 14
\end{aligned}
$$

Solve by the regular method and by Mayer's method. Some idea of their relative accuracies may be obtained by computing Σv^2 in each case.

METHOD BY DIMINISHING THE CONSTANT TERM

Suppose we have given

$$A+B=10$$
$$4A-B=19$$
$$2A+3B=25$$

An inspection of these equations shows that $A=6$ and $B=4$ approximately.

Let a and b equal the difference between the true and approximate values of A and B. Then

$$(6+a)+(4+b)-10=0$$
$$4(6+a)-(4+b)-19=0$$
$$2(6+a)+3(4+b)-25\ =0$$
$$a+b=0$$
$$4a-b+1=0$$
$$2a+3b-1=0$$

Forming normals and solving, we have

$$a=-0.153$$
$$b=+0.406$$
$$A=6-0.153=5.847$$
$$B=4+0.406=4.406$$

The student should compare these results with those obtained by the regular method.

This method is especially well adapted to the adjustment of angles observed at a station. Considerable

mathematical computation is avoided. The following problem from the United States Lake Survey is solved in Merriman's *Least Squares*.

No.	Between Stations.	Observation.
1.	Bunday and Wheatland............	44° 25′ 40″.613
2.	Bunday and Pittsford..............	80 47 32 .819
3.	Wheatland and Pittsford...........	36 21 51 .996
4.	Pittsford and Reading..............	91 34 24 .758
5.	Pittsford and Bunday..............	279 12 27 .619
6.	Reading and Quincy...............	62 37 43 .405
7.	Quincy and Bunday................	125 00 18 .808

We may set up the observation equations:

$$
\begin{aligned}
m_1 &= 44^\circ\ 25'\ 40''.613\\
m_1+m_3 &= 80\ \ 47\ \ 32\ .819\\
m_3 &= 36\ \ 21\ \ 51\ .996\\
m_4 &= 91\ \ 34\ \ 24\ .758\\
360^\circ-(m_1+m_3) &= 279\ \ 12\ \ 27\ .619\\
m_6 &= 62\ \ 37\ \ 43\ .405\\
360^\circ-(m_1+m_3+m_4+m_6) &= 125\ \ 00\ \ 18\ .808
\end{aligned}
$$

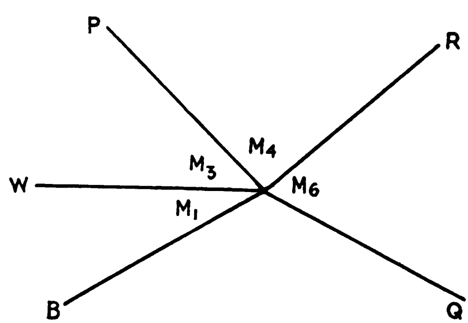

Fig. 6.

On account of the nature of the constant terms the solution would be extremely tedious. We may let v_1, v_3, v_4, and v_6 be the most probable corrections to be applied. Then

$$\begin{aligned} m_1 &= 44° \ 25' \ 40''.613+v_1 \\ m_3 &= 36 \quad 21 \quad 51 \ .996+v_3 \\ m_4 &= 91 \quad 34 \quad 24 \ .758+v_4 \\ m_6 &= 62 \quad 37 \quad 43 \ .405+v_6 \end{aligned}$$

This gives rise to simpler observation equations:

$$\begin{aligned} v_1 &= 0 \\ v_1+v_3 &= +0.210 \\ v_3 &= 0 \\ v_4 &= 0 \\ v_1+v_3 &= -0.228 \\ v_6 &= 0 \\ v_1+v_3+v_4+v_6 &= +0.420 \end{aligned}$$

The right-hand members denote seconds.

The normal equations are

$$\begin{aligned} 4v_1+3v_3+v_4+v_6 &= +0.402 \\ 3v_1+4v_3+v_4+v_6 &= +0.402 \\ v_1+v_3+2v_4+v_6 &= +0.420 \\ v_1+v_3+v_4+2v_6 &= +0.420 \end{aligned}$$

From which

$$\begin{aligned} v_1 &= +0''.022 \\ v_3 &= +0 \ .022 \\ v_4 &= +0 \ .126 \\ v_6 &= +0 \ .126 \end{aligned}$$

The adjusted values now become,

No. 1	44°	25′	40″	.635
2	80	47	32	.653
3	36	21	52	.018
4	91	34	24	.884
5	279	12	27	.347
6	62	37	43	.531
7	125	00	18	.932

CHAPTER IV

THE PRECISION OF OBSERVATIONS

As stated in the introductory chapter, there are no measurements made with such a degree of accuracy that we may regard them as absolutely correct. The wave-length of cadmium light has been measured with an accuracy which is marvelous, but its exact value will probably never be known. Since measurements differ among themselves in accuracy, it is desirable to have some method of indicating this fact, and the term "Precision of Observations" or "Precision of Measurements" is applied to the method of procedure employed in determining the relative accuracy of measurements. Strictly speaking, the term "deviation" is a better one than "accuracy" because the latter term seems to presuppose a knowledge of exact results.

The *percentage of error* is obtained by dividing the difference between the result obtained and the true result by the true result, and multiplying by 100. Here the "true result" may mean the best obtainable result.

The *percentage of deviation* is obtained by dividing the difference between two results, neither one of which may be regarded as necessarily correct, by either result, and multiplying by 100.

PROBLEMS

1. If a student brings a value for g of 981.7 cm. per sec. per sec. at a place where the best determinations give 980.6, what is his percentage of error? Would the term "deviation" apply here?

2. Find the percentage of deviation between two determinations of the density of mercury: 13.47 and 13.61.

Classes. There are three classes of precision measures:

1. Mean error;
2. Average deviation;
3. Probable error.

Graphic Method. These may be illustrated graphically by use of the probability curve (Fig. 7).

Since YOX may be regarded as a probability area, and since, as will be shown later, the probability that the true result lies within the limits indicated by the

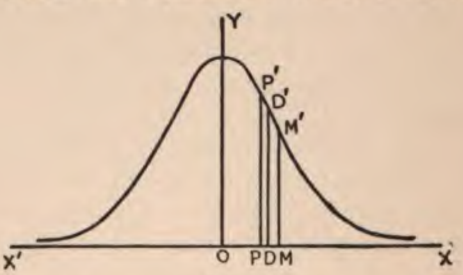

Fig. 7.

precision measure is one-half, it follows that we may express our precision in one of three ways:

1. By taking the distance from 0 along the x-axis to the point of inflection of the curve. This is represented by OM and expresses the mean error or mean square error.

2. By taking the abscissa of the ordinate passing through the center of gravity. This is OD and expresses the average deviation.

3. By taking the abscissa of the ordinate which divides YOX into equal parts. This is OP and expresses the probable error.

By reference to Problem 5, page 14, it will be seen that the mean error, $\mu = \frac{1}{h\sqrt{2}}$, the distance OM in the figure.

The average deviation, OD, may be found by use of the formula in mechanics

$$X = \frac{k\int_0^\infty xe^{-h^2x^2}dx}{k\int_0^\infty e^{-h^2x^2}dx} = \frac{1/2h^2}{\sqrt{\pi}/2h} = \frac{1}{h\sqrt{\pi}}.$$

The probable error, $OP = \frac{0.4769}{h}$. This will be deduced later.

Of these precision measures, it may be said that the mean error, "the square root of the arithmetical mean of the squares of the errors," is seldom used; the probable error is the most accurate; and the average deviation is the easiest to determine.

The Average Deviation

If m is the mean of n measurements and $v_1, v_2, v_3, \ldots v_n$ the residuals (neglecting signs), the average deviation would be $a.d. = \frac{\Sigma v}{n}$.

Taking the numbers 10, 8, 9, 10, the mean is 9.25, and $v_1 = 0.75$, $v_2 = 1.25$, $v_3 = 0.25$, and $v_4 = 0.75$. $\Sigma v = 3.00$ and $a.d. = 0.75$. We may neglect the signs of the residuals.

This is the average deviation of a single observation.

The average deviation of the mean, $A.D. = \frac{a.d.}{\sqrt{n}}$. In the problem just solved, $A.D. = \frac{0.75}{2} = 0.38$.

PROBLEMS

It will prove of interest if the instructor will assign problems taken from the student's laboratory note-book in illustration of this topic.

The Probable Error

The term probable error is rather misleading, but its meaning may be made clear by the following definition: The probable error is a number placed after a result with a plus and minus sign between them; and it indicates that it is an even wager that the true result lies between the indicated limits, and that it does not so lie.

At first thought, this definition seems vague and not indicative of a very high order of precision. Let us take the value of the velocity of light as 299860 ± 30 km. per second. This means that it is just as likely that the true value lies between 299830 and 299890 as that it lies between

zero and 299830, plus 299890 to infinity. This narrows the field and indicates an order of precision that is sufficiently high.

The probable error is sometimes defined as a quantity, which, when added to and subtracted from the mean, gives limiting values such that if another mean is determined in a similar manner, its value is as likely to lie outside these limits as to lie between them.

Formulae for Probable Error

Going back to the probability integral, we must take the value of x when $P=\frac{1}{2}$. This gives $\frac{1}{2}=2/\sqrt{\pi}\int_0^{hx} e^{-h^2x^2}dhx$.

From integration tables we find $hx=0.4769$.

The particular value of x which fulfills this condition will be denoted by r. Then $hr=0.4769$.

Take the equation for compound probability (page 15).

$$P=h^n dx^n \pi^{-n/2} e^{-h^2\Sigma x^2}.$$

Since h is the measure of precision, we wish to give it such a value as will render P a maximum.

$$\frac{dP}{dh}=nh^{n-1}e^{-h^2\Sigma x^2}-h^n e^{-h^2\Sigma x^2}2h\Sigma x^2=0$$

$$n-2h^2\Sigma x^2=0.$$

(Divide by $h^{n-1}e^{-h^2\Sigma x^2}$)

$$h=\sqrt{\frac{n}{2\Sigma x^2}}.$$

But $$h=\frac{0.4769}{r}.$$

Therefore $$r=0.4769\sqrt{\frac{2\Sigma x^2}{n}}=0.6745\sqrt{\frac{\Sigma x^2}{n}}.$$

This is the probable error for a single observation when *errors* are considered.

In order to change errors to residuals (x to v), we make use of the following procedure: Let the sum of the squares of the errors differ from the sum of the squares of the residuals by some constant, say u^2. Then

$$\Sigma x^2=\Sigma v^2+u^2.$$

Now $$u^2=\frac{\Sigma x^2}{n};$$

$$\Sigma x^2=\Sigma v^2+\frac{\Sigma x^2}{n};$$

$$(n-1)\Sigma x^2=n\Sigma v^2;$$

$$\frac{\Sigma x^2}{n}=\frac{\Sigma v^2}{n-1}.$$

Substituting in the formula for r,

$$r=0.6745\sqrt{\frac{\Sigma v^2}{n-1}}.$$

This is the formula for single observations for residuals.

Since probable errors are inversely proportional to the squares of the number of observations taken (see any text-book on Least Squares), we have

$$n : 1 :: 1/r_0^2 : 1/r^2,$$

$$r_0 = r/\sqrt{n}.$$

r_0 thus represents the probable error of the mean and may be written $r_0 = 0.6745\sqrt{\dfrac{\Sigma v^2}{n(n-1)}}$.

We may now summarize the relation between the probable error, the measure of precision, and the weight as follows:

1. The measure of precision varies inversely as the probable error (since hr = constant).

2. Weights are proportional to the squares of the precision measures.

3. Weights are inversely proportional to the squares of the probable errors.

PROBLEMS

1. Given the following measurements of the length of a line:

70.6 cm.	70.5 cm.	70.4 cm.
70.5 "	70.6 "	70.5 "
70.7 "	70.8 "	70.6 "

Find the mean square error, the average deviation, and the probable error.

2. Assume that the lines *OP*, *OD*, and *OM* are drawn to scale in Fig. 7, and compare their relative lengths with the results of No. 1.

3. Given the measurements and probable errors:

$$427.32 \pm 0.04$$
$$427.30 \pm 0.16.$$

Find the relative weight and the relative precision.

4. Twenty measurements of a line give a probable error of the mean of 0.06. How many additional measurements are required to reduce the probable error to 0.03?

5. An angle is measured 9 times with each of two transits. The first gives a value of

$$41^\circ\ 32'\ 14'' \pm 8''.2.$$

The second gives

$$41^\circ\ 32'\ 12'' \pm 7''.1.$$

Find the best value for the angle.

6. When the best value is found from No. 5, its probable error may be found by use of the formula

$$r_f^2 : r_a^2 :: p_a : p_f$$

where r_f = probable error in the best value;
r_a = probable error in the first value;
p_a = weight of first value;
p_f = weight of first value + weight of second value.

The precision measure is sometimes expressed as a fraction or a decimal with reference to the magnitude of the measurement concerned. Thus in Problem 3, we may say that 0.04 is the precision measure, or we may say that the measurement is reliable to about 0.01%, or to one part in 10,000.

PROBLEMS

1. Show from the data on page 37 that $r = 0.85\ a.d. = 0.67\mu$.

It follows that for a constant value of n, $r_0 = 0.85\ A.D.$

2. The following results have been obtained from nine measurements in each case:

94.31	cm.	$A.D. = 0.031$
94 36	"	$a.d. = 0.090$
94.35	"	$r = 0.025$
94.33	"	$r_0 = 0.007$
94.34	"	reliable to 0.03%
94.35	"	precise to 2 parts in 10,000

Reduce these different precision measures to *A.D.*'s and write them in order of the reliability of the results.

The fact that the size of the probable error must be considered in relation to the magnitude of the number with which it is written may be illustrated by Fletcher's result for the number of molecules in a gram-molecule of air: $N = 60.3 \times 10^{22} \pm 1.2 \times 10^{22}$. (*Phys. Rev.*, November, 1914.)

CHAPTER V

THE PROPAGATION OF ERRORS

IN the study of the propagation of errors we have two classes of problems—the direct and the converse. In the first class we determine how the errors in component measurements affect the reliability of the results; and in the converse problem we determine with what accuracy we should make our component measurements in order to secure a required accuracy in the result.

In solving problems under the first case, we proceed as follows: If Z represents the true result, and X its error; and $z_1, z_2, \ldots z_n$ are true values of component measurements, and $x_1, x_2, \ldots x_n$ are their corresponding errors, then,

$$Z+X=f[(z_1+x_1), \quad (z_2+x_2) \ldots (z_n+x_n)].$$

By Taylor's theorem

$$X=\frac{dZ}{dz_1}x_1+\frac{dZ}{dz_2}x_2+\ldots\frac{dZ}{dz_n}x_n.$$

Consider that we have a series of such terms, designated by ΣX, Σx_1, Σx_2, etc., and square both sides:

$$\Sigma X^2=\left(\frac{dZ}{dz_1}\right)^2\Sigma x_1^2+\left(\frac{dZ}{dz_2}\right)^2\Sigma x_2^2+\ldots\left(\frac{dZ}{dz_n}\right)^2\Sigma x_n^2.$$

We have neglected terms containing the products of the errors, such as $\Sigma x_1 x_2$, since positive and negative errors are equally likely to occur. If we divide both sides by n, we may substitute r^2 (the probable error in the final result) for $\frac{\Sigma X^2}{n}$ and r_1^2, r_2^2, r_n^2, the probable errors of the components for $\frac{\Sigma x_1^2}{n}$, etc. The reason for this is evident from an inspection of the probable error formula. Thus we have

$$r^2 = \left(\frac{dZ}{dz_1}\right)^2 r_1^2 + \left(\frac{dZ}{dz_2}\right)^2 r_2^2 + \ldots \left(\frac{dZ}{dz_n}\right)^2 r_n^2.$$

Hereafter we shall replace r by Δ and $r_1, r_2 \ldots r_n$ by $\delta_1, \delta_2, \ldots \delta_n$. These refer to any precision measure as well as the probable error. Δ means the deviation in the result due to deviations in the components, $\delta_1, \delta_2, \ldots \delta_n$.

ILLUSTRATIVE PROBLEMS

1. The two sides of a rectangle measure 45 ± 0.6 and 50 ± 0.4. Find the deviation in the area. The formula is $A = ab = 2250$.

$$\begin{aligned}
\Delta^2 &= \left(\frac{d(ab)}{da}\right)^2 (\delta_a)^2 + \left(\frac{d(ab)}{db}\right)^2 (\delta b)^2 \\
&= (50)^2(0.6)^2 + (45)^2(0.4)^2 \\
&= 900 + 324 = 1224 \\
\Delta &= 35 \text{ approximately.}
\end{aligned}$$

We may say, then, that the area (expressed in square feet), is 2250 with a deviation measure of 35.

2. It may be readily seen from the formula that if the equation has the form $A=a+b$, the value of Δ^2 becomes $\delta_a{}^2+\delta_b{}^2$. This may be applied to the measurement of a line which has to be made in sections.

3. If the equation contains a constant multiplier, $A=ca$, we have $\Delta=c\delta$.

If the radius of a circle is 10 cm. with a deviation of 0.1, the deviation of the resulting length of the circumference is $2\pi\times0.1\doteq0.63$.

PROBLEMS

1. The formula for the radius of a sphere as given by a spherometer is

$$R=\frac{l^2}{6a}+\frac{a}{2}.$$

If $$l=7.23\pm0.04 \text{ cm.},$$

and $$a=0.53\pm0.008 \text{ cm.}$$

find the length of the radius and its deviation measure.

2. If the length of a pendulum is 100 ± 0.1 cm., and the period of one vibration 1.01 ± 0.003 seconds, what is the deviation in g?

3. The mass of a body in air is 30 ± 0.1 g; in water 20 ± 0.2 g. Find the deviation in the value of its specific gravity.

4. What is the deviation in the value of the velocity of liquid flow from a height of 50 ± 1 cm.? $V^2=2gh$.

5. A mass of 100 g. is revolved with a speed of 80 cm. per second on the end of a cord 50 cm. long. The mass measurements are 101, 100, 99; the speed measurements

82, 80, 78; the length measurements 51, 49, 50. Find the centrifugal force and its deviation.

6. Find the deviation in the mean of two quantities which differ by C.

7. If d is the deviation in log a, what is the deviation in a?

8. In No. 2, assume that the deviation found is an average deviation, $A.D.$ Express the result fractionally, decimally, and as a probable error.

9. In determining a refractive index, the value of i is $40° \pm 8'$, and of r $32° \pm 6'$. Find the value of the index and its deviation.

In solving these problems, the student should observe the relative effects of the deviations in the components upon the result. In No. 2, for example, it will be seen that a deviation in t is a much more serious matter than an equal deviation in l. A large number of problems illustrating this process may be found in Goodwin's *Precision of Measurements.*

PROBLEMS

The following problems are of especial interest to engineering students:

1. In the triangle ABC, $AB = 500' \pm 0.04'$ $\angle A = 32° \pm 10'$, $\angle C = 60° \pm 4'$. Find BC and its deviation. The formula for the area of a triangle with the given conditions should be written down and the usual method employed. We have the deviations given for the angles and need them for the sines of the angles. For a deviation of 10 minutes in 32 degrees, we find the deviation in the sine of 32 degrees

by subtracting the sine of 32° from that of 32° 10′. Or we may differentiate with respect to the angle.

2. In a triangle ABC, we have $AB=240.4\pm0.04$, $AC=290.6\pm0.003$, $\angle A=44° 20'\pm1'$. Find the area and its deviation. Use both methods referred to above in dealing with the angle A.

3. The formula for the elastic modulus of a rectangular bar supported at its ends is $E=\frac{Pl^3}{4dbh^3}$, where P is the mass applied at the center, l the length, d the deflection produced, b and h the breadth and thickness. We have given the following measurements:

b	h	d
0″.331	0″.490	0″.206
0″.333	0″.492	0″.205
0″.329	0″.491	0″.206
0″.330	0″.490	0″.207

Take the length two feet and $P=40$ lbs.

Find E and its deviation.

4. Find the number of calories and its deviation, from a current of 6.2 ± 0.06 amperes, through a resistance of 20 ± 0.1 ohms, for 30 minutes (correct to a single second).

5. Observations on a resistance of about 10 ohms give values as follows: Correct to 0.09%, correct to one part in one hundred; a probable error of 0.001. Write these down in the order of their relative reliabilities.

An interesting case of the effect of errors in observed values upon derived values is found when the derived value is the difference of two observed values, both of which are large in comparison with their difference.

For example, the area of the datum circle of a given plan-

imeter is 2075 sq. cm. A student by measurement obtains the value of 2070 sq. cm., which is correct to less than one-fourth of one per cent. The student then traces a given figure with the fixed point at the center of the figure and the movable point moving in a counter-clockwise direction, giving the reading 2010 sq. cm. The area of the figure is, then, either $2075-2010=65$ sq. cm., or $2070-2010=60$ sq. cm., results which differ from each other by about eight per cent.

THE CONVERSE PROBLEM

In the cases already considered, we have discussed the effects of deviations in component measurements upon the deviation in the result. We are commonly called upon to determine in advance to what degree of precision we shall make our measurements in order to reach a given precision in our result. For example, we are required to measure the volume of a cylinder whose length is about 10 cm. and whose radius is about 3 cm. with such a degree of accuracy that the deviation in the volume (about 283 cm.^{3}) shall not exceed 3 cm.^{3} It is customary to so adjust the deviations pertaining to each variable that each will have an equal effect upon the final result. This is called the method of *equal effects*. Representing the effects of the various deviations upon the results by Δ_a, Δ_b, . . . Δ_n, we have $\Delta^2=\Delta_a{}^2+\Delta_b{}^2+\ .\ .\ .\ \Delta_n{}^2=n\Delta_a{}^2$. $\Delta=\Delta_a\sqrt{n}$. In our problem we may write

$$\Delta_l=\Delta_r=\frac{\Delta}{\sqrt{n}}=\frac{3}{\sqrt{2}}=2.1\ \mathrm{cm}^3.$$

The deviations in the length and radius should not separately produce a deviation in the area of more than 2.1 cm.[3] From our general equation, page 45, we have

$$\Delta_l = \frac{dA}{dl}\,\delta l = \pi r^2\,\delta l = 28.3\,\delta l = 2.1.$$

Therefore $\delta l = 0.074$ cm. Likewise

$$\Delta_r = \frac{dA}{dr}\,\delta_r = 2\pi r l\,\delta_r = 188.6\,\delta_r = 2.1. \quad \delta_r = 0.011 \text{ cm.}$$

We conclude that we must measure the length of the cylinder with an accuracy of 0.074 cm., and the radius with an accuracy of 0.011 cm.

The Fractional Method

These problems may be solved in another way. It follows (not quite directly) from the method of equal effects that the deviation in a final result due to a deviation in a component bears the same relation to the final result as the deviation in the component bears to the component, if the exponent is unity.

In other words $\frac{\Delta_l}{A} = \frac{\delta_l}{l}$. If the component has any other exponent than unity, the fraction must be multiplied by the exponent, following the method of differentiation. We have in the last problem $\frac{\Delta_r}{A} = \frac{2\,\delta_r}{r}$, since the radius appears in the second power.

Since $\Delta_l = \Delta_r = \frac{\Delta}{\sqrt{2}}$ we may write

$$\frac{\Delta_l}{A}=\frac{\Delta_r}{A}=\frac{1}{\sqrt{2}}\frac{\Delta}{A}=\frac{1}{\sqrt{2}}\frac{3}{283}\doteq 0.0074$$

$$\frac{\delta_l}{l}=0.0074;\quad \delta l=0.074$$

$$\frac{\delta_r}{r}=\frac{1}{2}\times 0.0074=0.0037;\quad \delta_r=0.011.$$

This is seen to check with the former method.

PROBLEMS

1. With what accuracy must we measure the radius of a circle to obtain an accuracy of 0.1% in the area? (First method.)

2. With what accuracy should l and t be measured in a simple pendulum to obtain a value of g correct to one unit? (Second method.)

3. With what accuracy should h and a be measured with a spherometer to give a value in the radius correct to 0.03%? This problem may be solved in a literal form or values may be assigned to a and b and the result computed. (First or second method.)

These are illustrative problems. They should be extended, so far as time permits, to include various problems from the student's laboratory note-book. The importance of this part of the subject is obvious.

BEST MAGNITUDES AND BEST RATIOS

No one can work very long in a physical laboratory without observing that the results come out much better by using certain magnitudes and ratios than they do with others. In measuring current with a tangent galvanometer very poor results would be obtained for angles between 0°–20° and 70°–90°. Around 45°, however, they are satisfactory. In using a slide-wire Wheatstone bridge, the two segments of the wire must be about equal for satisfactory results.

In the first case, our formula is $I = c \tan \phi$.

$$\Delta_\phi = \frac{d(c \tan \phi)}{d_\phi} \delta_\phi = \frac{c\,\delta_\phi}{\cos^2 \phi}.$$

Dividing

$$\frac{\Delta_\phi}{I} = \frac{c\,\delta_\phi}{\cos^2 \phi} \times \frac{1}{c \tan \phi} = \frac{2\,\delta_\phi}{\sin 2\phi}.$$

This is a minimum when $\phi = 45°$ and shows that the deviation in the result due to a deviation in ϕ is least at 45°.

For the Wheatstone bridge we may let $x =$ the unknown resistance, R the known, a and b the segments of the wire, and c its total length.

Then

$$x = \frac{aR}{c-a}.$$

It should be easy for the student to prove that the best result occurs when $a=b$.

These illustrations might be indefinitely extended. The student is referred to Holman's *Precision of Measurements* for a number of interesting problems.

EXERCISES

1. Show that the relative error in measuring the area of a circle decreases with increasing radius.

2. In drawing a simple harmonic motion curve, discuss the effect of an error in the phase upon the displacement for various points on the curve.

3. A body rolls down an inclined plane for a definite period of time. Discuss the effect of an error in measuring the angle of the plane upon the error in velocity. Neglect friction.

4. How would an error of one minute in measuring the angle of incidence compare with a similar error in measuring the angle of refraction in their effect upon the refractive index? Take $i=45°$ and $r=30°$.

5. What is the "best value" of the limiting angle in measuring the coefficient of friction?

6. In Lami's theorem $\dfrac{A}{\sin\alpha}=\dfrac{B}{\sin\beta}=\dfrac{C}{\sin\gamma}$,

take
$$A=59.8,\quad B=69.8,\quad C=102.8,$$
$$\alpha=146°,\quad \beta=139°,\quad \gamma=75°.$$

Discuss the effect upon each term of an error of $0°.5$ in measuring α, β, and γ.

CHAPTER VI

PLOTTING

Definitions. It may be assumed that students for whom this book was prepared have a knowledge of the fundamental principles of curve tracing. A few familiar definitions and illustrations will, however, be given.

A *plot* is a graphic representation of the relation of two quantities of which one is a function of the other. The *origin* is the point of departure from which all distances are reckoned. The *coördinates* are horizontal and vertical distances from the origin. The axis of *abscissas* is the horizontal line through the origin. The axis of *ordinates* is the vertical line through the origin. The *slope* of the curve is the angle it makes with the x-axis.

When the points for a curve have been established, there are two methods of procedure: If the curve appears to be regular or to correspond with well-known types, we should draw an average line through the points in such a manner that about as many will lie on one side as on the other. If the curve does not fulfill these conditions, the adjacent points should be connected by straight lines.

EXERCISES

In these exercises, the student should use his judgment as to the method to be followed.

1. Plot a simple interest curve for $100 for ten years at 5 per cent.

2. Plot a compound interest curve starting from the same origin.

3. The maximum temperature for each day of a certain month was as follows:

1	31°	11	29°	21	11°
2	22	12	24	22	13
3	21	13	18	23	18
4	27	14	12	24	41
5	29	15	14	25	43
6	26	16	26	26	38
7	22	17	28	27	30
8	28	18	28	28	36
9	29	19	19	29	36
10	28	20	16	30	47

Plot the curve.

4. Compute the mean temperature for the month in No. 3, and plot a curve showing the departure from the mean for each day.

5. The following plot will afford amusement as well as instruction:

x	y	x	y
−0.0	1.5	1.3	−1.0
−1.0	1.2	1.6	−0.3
−2.0	0.7	2.0	−0.4
−3.0	0.0	3.0	−0.5
−3.5	−0.5	3.3	−0.5
−3.7	−1.0	3.5	−0.3
−3.5	−1.7	3.3	0.0
−3.0	−2.2	3.0	0.6
−2.5	−2.3	3.0	1.0
−1.0	−2.1	2.7	1.3
−0.5	−1.9	2.8	1.6
0.0	−2.3	2.5	1.5
0.5	−2.4	2.0	1.4
1.0	−2.5	1.0	1.6
0.8	−1.6	0.0	1.5
1.0	−1.3		

6. A thermometer is found to have the following errors when calibrated by means of a standard:

$$\begin{array}{ll} 10° & +0.03 \\ 12 & -0.60 \\ 18 & -0.45 \\ 22 & -0.38 \\ 30 & +0.71 \end{array}$$

Draw a curve of errors for the thermometer.

Determination of Constants. Let us take the equation of a straight line,

$$y = ax + b.$$

By definition $\frac{dy}{dx}$ is the tangent of the angle which the

line makes with the x-axis. This is equal to a and determines this constant. When $x=0$, $y=b$. This determines the constant b.

Rearrangement of Data. It sometimes happens that if a curve is plotted in one way it is meaningless or difficult to interpret; while a rearrangement of data makes it perfectly intelligible. If, for example, a series of readings have been taken with a Boyle's law apparatus, the resulting curve should be an equilateral hyperbola ($pv=a$). It will be found, however, that the readings one is ordinarily able to obtain are not sufficient to identify the curve. If we change the data and plot v and $1/p$ we get a straight line.

In a tangent galvanometer the formula is $I=k \tan \phi$. In order to get a series of variations in the current we introduce varying resistances. Since $I=E/R$, it follows that the reciprocals of the resistances will plot a straight line with $\tan \phi$

EXERCISES

1. The following data come from an experiment with the Boyle's law apparatus:

Volume.	Pressure.	Volume.	Pressure.
26.5	134.3	34.9	102.9
27.4	130.5	36.2	99.3
27.9	126.4	37.6	95.7
28.2	122.9	39.2	92.3
29.2	118.4	40.7	88.3
31.3	114.5	42.5	85.3
32.4	110.5	44.0	81.9
33.6	106.7	45.8	78.9

Discuss the accuracy of the observations from the plot.

2. The following table is made up from resistances and corresponding deflections with a tangent galvanometer:

Resistances (Ohms).	Deflections (Degrees).
90	5.3
50	8.7
30	12.8
20	16.7
10	24.0
5	30.6
3	34.5
1	39.3
0	42.5

a. Assume that we have a constant electromotive force and make a plot which will show the relation between the current and the deflection.

$$I = k \tan \theta, \qquad I = E/R.$$

b. Plot the resistances with the cotangents of the angles and project until it cuts the x-axis. Interpret the curve.

c. Find the value of E by Ohm's law.

3. Known volumes of a liquid were placed in a flask and weighed, giving data as follows:

Volume of Liquid. cc.	Total Mass (Liquid + Flask.) g.
20.8	204
37.6	221
61.3	250
86.5	287
108.0	307
136.5	336

Plot the volume on the x-axis and the masses on the y-axis.

a. Find the mass of the flask from the plot.

b. Find the specific gravity of the liquid.

4. Two rulers were placed together at random and the reading on the centimeter scale was taken opposite each inch division on the English scale, giving data:

Inches.	Centimeters.
0	——
1	——
2	27.8
3	25.3
4	22.7
5	20.2
6	17.7
7	15.1
8	12.6
9	10.0
10	7.5
11	5.0
12	2.4

Plot the inch readings on the x-axis and the centimeter readings on the *y-axis*.

a. Determine from the plot how the rulers were placed with reference to each other.

b. Find the ratio of the centimeter and inch by using the intercepts of the curve on the axes; also by using the ratio of the difference of two points y_2-y_1 and x_2-x_1.

5. The candle-power of a 16-c.p., 110-v. lamp was measured for different voltages across the terminals as follows:

Volts.	Candle-power.
60	0.4
70	1.1
80	2.3
90	4.2
100	8.3
105	10.3
118	16.6
128	22.3
138	25.8
150	44.7
160	54.5
170	72.2
180	79.7
190	93.9
200	115.7
210	120.3

Plot volts on x and candle-powers on y-axis.

a. Find candle-power at 110 v.

b. Find candle-power at 220 v.

c. Is it possible to estimate the voltage at which the filament first begins to glow?

6. The elongation of a spring was measured for different loads, and the energy stored in the spring computed, giving data:

Loads.	Elongation.	Energy.
g.	cm.	g. cm.
0	0.0	0
50	2.3	57.5
100	5.4	270.0
150	8.8	660.0
200	11.9	1190.0
250	15.2	1900.0

Plot loads (x) with elongations (y) and energy (y).

a. Compute the constant of the spring (g/cm).

b. We have

$$W = Fd = \left(\frac{O+L}{2}\right)E;$$

$$E = kL;$$

$$W = \frac{kL^2}{2}.$$

Interpret this equation and compare with the curve.

c. Compute the area between the load-elongation curve and the x-axis. What does this represent?

The electrical resistance of lead at various temperatures indicated by means of a plot that the resistance would be zero at $-273°$ C. The recent experiments of Professor Kamerlingh Onnes have justified this conclusion.

7

Inches.	Centimeters.	Weight Given.
0	0	
1	2.5	10
2	5.5	1
3	7.3	3
4	10.0	10
5	13.3	2
6	14.8	8
7	18.2	8
8	19.0	6
9	23.6	5
10	24.4	1

Plot the above, taking inches on the x-axis and centi-

meters on the y-axis. Draw an average straight line through the points and test its accuracy by reference to the equation $\Sigma r = 0$, when the various perpendicular distances from the points to the line are expressed by r_1, r_2, etc. Repeat the exercise, using the weights assigned above. Correct the line by use of the results obtained.

CHAPTER VII

NEGLIGIBILITY

Importance. It is of the utmost importance that students in laboratory courses should come to know under just what conditions they are at liberty to neglect small quantities which appear in their work. A student who comes into physics from courses in mathematics is conscious of a decided shock when he is told to throw away certain terms in an equation. It is hoped that a few illustrations will convince him that such a process is not at all unscientific or inaccurate.

We learn in trigonometry that the sine, tangent, and radian value of an angle may be used interchangeably if the angle is small. The following table will make this clear:

Degrees.	Radians.	Sine.	Tangent.
0°	0	0	0
0 30′	0.00873	0.00873	0.00873
1	0.01745	0.01745	0.01746
2	0.03491	0.03490	0.03492
3	0.05236	0.05234	0.05241
4	0.06981	0.06976	0.06993
5	0.08727	0.08716	0.08749

It will be seen that for most purposes very little error would result from substituting the radian measure or the tangent for the sine.

EXERCISES

1. Construct a circle with radius unity. Take an angle of 60° at the center and draw an arc and a chord. Call the arc dx and the chord dy (equal to the radius). Find the value of the arc by radians $\left(\frac{2\pi}{6}\right)$, and the chord by the law of sines. In this case $dy=1$ and $dx=1.0472$. Find relation between dx and dy for 30°, 10°, 5°, and 1°.

2. What is the largest angle for which $dx=dy$ to four places of decimals?

The Pendulum Formula. In deducing the formula for the simple pendulum, we make use of an approximation, so that the equation $T=2\pi\sqrt{l/g}$ is not quite correct. The equation

$$T=2\pi\sqrt{l/g}\left[1+\left(\frac{1}{2}\right)^2K^2+\left(\frac{1.3}{2.4}\right)^2K^4+\ldots\right]$$

tends to diminish the error as more and more terms are used. Here $K=\frac{\theta}{2}$.

EXERCISE

Give θ values of 60°, 10°, and 1° and assume that the longer formula gives the correct value of g. Find the error in each case due to using the shorter formula. Remember that g appears as a square root.

The Mirror Formula. In developing the formula for the circular mirror a similar approximation is noted. By using a parabolic mirror no approximation appears.

EXERCISES

1. Show that light starting from the focus of a paraboloid of revolution will go in parallel lines upon reflection from any part of the mirror.

2. Construct a parabola whose equation is $y^2=4px$. Construct a circle internally tangent to this whose equation is $x^2+y^2-2rx=0$. Let $r=2p$. Select points along the axis distant $r/4$, $r/2$, $3r/4$, and r from the origin. Find the value of y on the circle and the corresponding value of y on the parabola in each case. The relation of these values may be used to measure the error due to the approximation.

3. Find the angular aperture at the center of the circle for each point taken.

Approximate Squares and Square Roots. There is a short method of squaring numbers and extracting their square roots which involves an approximation. Let us take the number 1.01 and consider it made up of $1.0+0.01$. If we square this by use of the binomial theorem, we have

$$1+0.02+0.0001=1.0201.$$

This is nearly equal to 1.02. If, therefore, we square our first term and add twice the second term, we have a rule for squaring such numbers as these. The square of 1.0019 is 1.0038036, to which 1.0038 is a close approxi-

mation. If the whole number is any other than unity, its value must be multiplied into the smaller term.

By a similar process we may find the square root of 1.04 to be 1.02, and that of 4.04 to be 2.01.

The Slide Rule

The slide rule is an instrument that gives approximate values and well illustrates the principle of negligibility. The following table shows its accuracy for certain processes:

Problem.	Solution by Slide Rule.	Solution by Five Place Logs.	Deviation, %
$152\times391\times31$	1842000	1842400	0.022
$86.4\times0.028\times4.95$	11.97	11.975	0.042
$496\div381$	1.303	1.3018	0.092
$292\div468$	0.624	0.62394	0.0096
$\dfrac{25\times0.07\times1.15}{378\times0.98}$	0.00543	0.0054327	0.05

The student should note the differences in the % of deviation.

The Value of π. Much energy has been expended in obtaining values of π correct to a large number of decimal places. It has been carried out to 707 places. (See *Seven Follies of Science* by Phin.) If π is carried out to six or seven places, it is sufficient for all purposes. The value $3\frac{1}{7}$ will answer for the majority of cases. It will prove convenient to remember that $\pi^2=9.87$ to a very close approximation.

PROBLEMS

1. Find the error due to using the value $3\frac{1}{7}$ carried out to four decimal places, for π.

2. Find the error due to using the value 9.87 for π^2, when π is carried out to four places.

3. We are accustomed to assume in physics that the coefficient of cubical expansion of a solid is three times that of the linear expansion. What error does this involve when a bar of brass, 100 cm. long at 0° C., is heated to 100° C.?

The error involved in using a mirror and scale (Poggendorff's method) is discussed in Stewart and Gee's *Practical Physics*, Vol. I, p. 55.

CRITERIA FOR NEGLIBILITY

It often happens that in a series of measurements of the same quantity, there are one or more values which do not compare well with the others. The question arises, what shall be done with these observations? They seem to have been taken with equal care with the others, and there seems to be no particular reason for rejecting them. Several criteria have been proposed for testing such observations.

The Huge Error. We find the mean and average deviation (*a.d.*) omitting the doubtful observation; then find the difference between the doubtful observation and the mean, and if this equals or is greater than 4 *a.d.*, it should be rejected.

Suppose we have a set of measurements: 10, 9, 10, 9, 12. The last measurement is questionable. The mean

is 9.5. $a.d.=0.5$. $12-9.5=2.5$. $4\times0.5=2.0$. The measurement should be rejected.

PROBLEM

The following length measurements (centimeters) have been taken: 60.1, 60.2, 60.3, 59.9, 58.3, 63.1. Apply the method to the last two results.

Chauvenet's Criterion. This is a more elaborate method. We may let t stand for the ratio between the limiting error (x) and the probable error of a single observation (r). If there are n errors, we may assume that nP will be the number less than x, and $n-nP$ the number greater than x.

Then, by definition

$$n-nP=\tfrac{1}{2}.$$

$$P=\frac{2n-1}{2n}.$$

We have

$$P=\frac{2}{\sqrt{\pi}}\int_0^t e^{-t^2}dt. \quad \text{(See page 15.)}$$

If we equate these two values of P and solve, we may find a value of t corresponding to any value of n. A few common values of n are given:

n	t	n	t	n	t
3	2.05	7	2.67	11	2.96
4	2.27	8	2.76	12	3.02
5	2.44	9	2.84	13	3.07
6	2.57	10	2.91	14	3.12

Since the limiting error, $x=tr$, we have only to find these two values and compare.

Let us take the following data: 12, 13, 12, 13, 13, 15. The mean is 13 and

$$\Sigma v^2=6. \quad r=0.6745\sqrt{\frac{\Sigma v^2}{n-1}}=0.6745\sqrt{\frac{6}{5}}=0.74.$$

For $\qquad n=6, \quad t=2.57, \quad x=tr=1.9.$

The deviation of the term in question is 2.0 and it should be rejected.

PROBLEM

The following observations were taken with a transit,

$$\begin{array}{r}
43^\circ\ 52'\ 26''.4 \\
28\ .5 \\
27\ .8 \\
28\ .0 \\
31\ .3 \\
30\ .2 \\
27\ .9
\end{array}$$

Test such observations as may be necessary by Chauvenet's Criterion.

Criterion for Negligibility for Deviation in Components

When the deviation in any component contributes but little to the deviation in the result, it is sometimes proper to neglect it altogether. We know that the error in the final result,

$$\Delta=\sqrt{\Delta_1^2+\Delta_2^2+\ .\ .\ .\ \Delta_v^2}$$

Suppose Δ_2 is the quantity under investigation. Let

$$\Delta_0=\sqrt{\Delta_1^2+\ .\ .\ .\ \Delta_n^2},$$

where Δ_2^2 has been omitted.

$\Delta-\Delta_0=$ the reduction in the deviation in the result due to omitting Δ_2. We may assume this value to be anything we please, depending upon the accuracy we require. If we insist, as some authors suggest, that it be equal to or less than $\frac{1}{10}\Delta$, we have

$$\Delta_0 \overline{\overline{>}} 0.9\Delta.$$

But $\quad \Delta_2^2=\Delta^2-\Delta_0^2=\Delta^2(1-0.9^2)=0.19\Delta^2.$

$$\Delta_2=0.43\Delta.$$

We may thus neglect a deviation if it contributes less than 0.43 of the total deviation of the result. A more rigid criterion could, of course, be determined by substituting a smaller value for the $\frac{1}{10}$.

Significant Figures. This is a subject that may be properly treated under the head of negligibility. At each stage in a series of measurements the student should look over his work and eliminate such figures as lend nothing to its precision. A rather full treatment of this subject is given in Holman's *Precision of Measurements,* to which the student is referred.

The following rules of precision are given:

1. When a rejected figure is five or over, increase the previous figure by one.

If we decide to drop the 7 in 14.637, we should write it 14.64.

2. In the deviation measure, we should retain two significant figures. Thus $a.d. = 0.062$; $r = 1.6$.

3. In our measured quantity, we should retain places corresponding to the second significant figure in the deviation measure.

$$m = 368.731 \pm 0.21 \text{ becomes } 368.73,$$
$$m = 406.67 \pm 2.6 \text{ becomes } 406.7.$$

4. In adding quantities, retain such places as correspond to the number having the largest deviation measure.

Quantity.	*a.d.*	Result.
374.20	0.36	374.20
4768.121	0.012	4768.12
984.1698	0.0021	984.17
		6126.49

5. In multiplication and division, find the quantity whose $\frac{a.d.}{m}$ is the greatest. From this find the percentage precision and if this is 1 per cent or more, use four significant figures; if between 1 and 0.1 per cent use five significant figures; if between 0.1 and 0.01 per cent, use six significant figures.

If we have to multiply 41.6 ± 0.4 by 590.25 ± 0.06, we note that $\frac{a.d.}{m}$ is greater for the first than for the second. As this is less than 1 per cent, we should retain five significant figures, giving 24556 as a result.

PROBLEMS

1. We have given the number 504.628 with the following deviation measures: $A.D.=0.21$; $r=0.031$; correct to one part in one hundred; correct to two per cent. What is the proper expression for the number in each case?

2. Add

$$\begin{array}{l} 21.42 \ \pm 0.61 \\ 338.161 \pm 0.042 \\ 543.1 \ \ \pm 1.5 \end{array}$$

3. Multiply

$$630.45 \pm 0.62 \text{ by } 25.635 \pm 0.024.$$

CHAPTER VIII

EMPIRICAL FORMULAE AND CONSTANTS

Definitions. A mathematical formula is one that is deduced by a process of reasoning along mathematical lines. The formula for the distance passed over t by a falling body is an illustration.

$S=v_0t+\frac{1}{2}at^2$ is derived from certain definitions of velocity and acceleration, to which mathematical processes have been applied.

An empirical formula cannot be derived in this manner, but depends upon the results of experiments which are treated in a manner to be described. The following is a typical empirical formula:

$$g=980.6056-2.5028 \cos 2l-0.000003h.$$

This has been made up from a large number of measurements of g under various conditions of latitude (l) and altitude (h).

In order to explain the method employed in constructing empirical formulae, let us take an experiment illustrating the relation of the space passed over by a falling body, to the time of fall.

The following may be assumed to be the results of the experiment:

t	s
1	1
2	4
3	9
4	16

The obvious relation existing between t and s will be disregarded, and we will endeavor to find an empirical

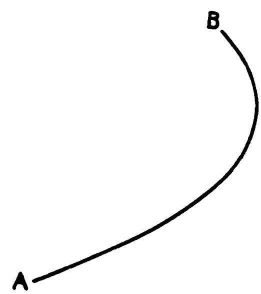

FIG. 8.

formula which fits the conditions. Our first step is to plot a curve.

This is seen to have the characteristics of a parabola and we write the general equation,

$$y = S + Tx + Ux^2 +, \text{ etc.}$$

x has been substituted for t, and y for s. From this are found observation equations:

$$\begin{aligned}1&=S+T+U\\4&=S+2T+4U\\9&=S+3T+9U\\16&=S+4T+16U\end{aligned}$$

The normal equations are:

$$\begin{aligned}30&=4S+10T+30U\\100&=10S+30T+100U\\354&=30S+100T+354U\end{aligned}$$

When these equations are solved, we have

$$\begin{aligned}U&=1\\T&=0\\S&=0\end{aligned}$$

and our equation becomes $y=x^2$.

If the experimenter had used only three seconds and had recorded an 8 instead of a 9, the values would have been

$$\begin{aligned}U&=0.5\\T&=1.5\\S&=-1.0\end{aligned}$$

Giving $$y=-1+1.5x+0.5x^2.$$

Upon repeating the experiment as the space measurement approaches closer to 9 this formula approaches the value $y=x^2$.

While no one would think of actually applying this method to so simple a case, it affords a good illustration of the manner in which all empirical formulae are constructed.

Classes of Curves. The following are some of the common curves from which these formulae may be constructed, together with their formulae and illustrations:

1. Straight line. $x=y$.
2. Parabolic. $y=S+Tx+Ux+$, etc.
3. Cyclic. $y=S+T\sin\frac{2\pi}{m}x+T'\cos\frac{2\pi}{m}x+$, etc.
4. Logarithmic. $y=be^{ax}$.
5. Hyperbolic. $xy=a$.

The formula for velocity due to gravity, $v=gt$, illustrates the first class. The space passed over by a falling body, the change in velocity of a river below its surface, and the growth of the United States in population, illustrate the second class. Cyclic curves may be used to represent the rise and fall of temperature, pressure, and humidity through a given interval. The logarithmic curve represents plots made from Newton's law of cooling, the absorption of light for varying thicknesses, and the gain due to compound interest. A familiar example of a hyperbolic curve is afforded by Boyle's law, $pv=ct$. This will plot a rectangular hyperbola.

Rules. The method of procedure may be summarized in the following rules:

1. Write the observation equation.
2. Plot the curve.

3. Identify the curve and write its equation.
4. Form the normal equations.
5. Solve for the unknowns, and these give the values of the constants sought.

It will be observed that every additional observation taken alters the values of the constants.

EXERCISES AND PROBLEMS

For the straight line and rectangular hyperbola an inspection of the data is generally sufficient to determine the value of the constant.

For the parabola the following examples should be studied:

1. Vertical velocity curve on the Mississippi river.

Depth.	Velocity.
0.0 depth	3.1950 feet per second
0.1	3.2299
0.2	3.2532
0.3	3.2611
0.4	3.2516
0.5	3.2282
0.6	3.1807
0.7	3.1266
0.8	3.0594
0.9	2.9759

The curve is obviously a parabola.

The first two observation equations are

$$3.1950 = S + 0.0T + 0.00U,$$
$$3.2299 = S + 0.1T + 0.01U.$$

Write down the remaining eight observation equations, form the normals, and solve. The results are

$$S = 3.19513$$
$$T = 0.44253$$
$$U = -0.7653, \text{ giving the formula}$$
$$Y = 3.19513 + 0.44253X - 0.7653X^2.$$

FIG. 9.

It will prove of interest to determine the velocity for each depth and compare with that obtained by experiment. (Why should they fail to check?)

2. The growth in the population of the United States is a good illustration of this curve, although from the nature of the case the results are not very reliable. In *Popular Science Monthly* for April, 1910, page 382, a set of curves are drawn for various countries. It will be seen that Sweden and Norway, Turkey, Spain, and Italy may be fairly represented by straight lines. The United States shows an

easily recognized parabola. In the article referred to the complete solution of the census problem is given.

3. The following illustrates a cyclic curve:

θ	Y	θ	Y
0°	20	200°	14
20	38	220	−13
40	51	240	−24
60	64	260	−30
80	68	280	−28
100	70	300	−24
120	64	320	−13
140	53	340	− 2
160	16	360	20
180	20		

Substitute θ for $\frac{2\pi}{m}x$ in the equation and plot the curve. Use the first three terms and form an observation equation for each value of θ. We have for the first four observation equations:

$$20 = S + T \sin\ 0° + T'$$
$$38 = S + T \sin 20° + T' \cos 20°$$
$$51 = S + T \sin 40° + T' \cos 40°$$
$$64 = S + T \sin 60° + T' \cos 60°$$

Normal equations for S, T, and T' are formed from these and their solution supplies the values of the constants in the formula.

4. We may use Winkelmann's data showing the rela-

tion between the temperature of a cooling body at different times, as an illustration of a logarithmic curve.

θ	t_2-t_1
18.9	3.45
16.9	10.85
14.9	19.30
12.9	28.80
10.9	40.10
8.9	53.75
6.9	70.95

The logarithmic relations of these quantities is not obvious by direct inspection. It is reached by the following process: We assume that the ratio at which a body loses heat $(-dQ)$ is proportional to the difference between its temperature and that of its surroundings. This is expressed by

$$-\frac{dQ}{dt}=k(\theta-\theta_0).$$

By definition of specific heat

$$Q=sd\theta.$$

Substitute this in the equation above

$$-\frac{d\theta}{dt}=a\theta, \text{ where } a=\frac{k}{s} \text{ and } \theta_0=0.$$

From this we get by integrating

$$\log b-\log\theta=at.$$

Now if θ_1 represents temperature at time t_1, and θ_2 temperature at time t_2, we have

$$\log b - \log \theta_1 = at_1;$$

$$\log b - \log \theta_2 = at_2;$$

From which $$a = \frac{1}{t_2 - t_1} \log \frac{\theta_2}{\theta_1}.$$

Take $\theta_2 = 19.9$ and substitute the values of θ for θ_1 and show that a is a constant, approximately equal to 0.0065.

Such an equation as $-\frac{d\theta}{dt} = a\theta$, where the change in a quantity is proportional to the quantity itself, is called a "compound interest equation."

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